Summation 1

14 January 2024 11:31

Swapping order of Summation :-

Let $f(a,b)$ be a function. Then

$$\sum_{a\in A}\sum_{b\in B} f = \sum_{b\in B}\sum_{a\in A} f$$

Changing Variables :-

$$\sum_{a\geq 0}\sum_{b\geq 0} f = \sum_{k\geq 0}\sum_{\substack{a,b\geq 0\\ a+b=k}} f$$

For every $a\geq 0$ we must add f for all $b\geq 0$

For every $k\geq 0$ add f for all $a,b\geq 0$ where $a+b=k$

$a+b=k$ means for $a=c\geq 0$ we have $b=k-c\geq 0$ is also arbitrary
c is arbitrary and as k is independent of a and b, $k-c$
which is just the LHS.

• Let us try to factorize,

$$\sum_{a\in A}\left(\sum_{b\in B} a^2(b+1)\right)$$

$$= \sum_{a\in A} a^2\left(\sum_{b\in B}(b+1)\right)$$

$$= \left(\sum_{b\in B}(b+1)\right)\left(\sum_{a\in A} a^2\right)$$

Q) Evaluate the sum,

$$\sum_{n\geq 1}\left(\left(\frac{7n+32}{n(n+2)}\right)\left(\frac{3}{4}\right)^n\right)$$

Ans:- $\frac{7n+32}{n(n+2)} = \frac{16n+32-9n}{n(n+2)} = \frac{16(n+2)}{n(n+2)} - \frac{9n}{n(n+2)}$

$$= \frac{16}{n} - \frac{9}{n+2}$$

$$\sum_{n\geq 1}\left(\left(\frac{16}{n} - \frac{9}{n+2}\right)\left(\frac{3}{4}\right)^n\right)$$

$$= \sum_{n\geq 1}\left(\frac{16}{n}\left(\frac{3}{4}\right)^n - \frac{9}{n+2}\left(\frac{3}{4}\right)^n\right)$$

$$= \sum_{n\geq 1}\left(\frac{16}{n}\left(\frac{3}{4}\right)^n - \frac{9}{n+2}\left(\frac{16}{9}\right)\left(\frac{3}{4}\right)^{n+2}\right)$$

$$= \sum_{n\geq 1}\left(\frac{16}{n}\left(\frac{3}{4}\right)^n - \frac{16}{n+2}\left(\frac{3}{4}\right)^{n+2}\right)$$

$$\sum_{n\geq 1}\frac{16}{n+2}\left(\frac{3}{4}\right)^{n+2} \equiv$$

$$= \frac{16}{1}\left(\frac{3}{4}\right)^1 + \frac{16}{2}\left(\frac{3}{4}\right)^2 + \cancel{\sum_{n\geq 2}\left(\frac{16}{n}\left(\frac{3}{4}\right)^n\right)} - \cancel{\sum_{n\geq 2}\left(\frac{16}{n}\left(\frac{3}{4}\right)^n\right)}$$

$$= 12 + \frac{9}{2} = \frac{33}{2}$$

Homework

Q> $1 + \frac{1}{4}\sqrt{2} + \frac{1}{4} + \frac{1}{16}\sqrt{2} + \frac{1}{16} + \frac{1}{64}\sqrt{2} + \cdots\cdots \infty$ terms. Evaluate

Q> Let $A = \underbrace{666\ldots666}_{2016\text{ times}}$ and $B = \underbrace{999\ldots999}_{2016\text{ times}}$. Let $N = A\times B$

Find the sum of digits of N.
